

A Novel Sufficient Condition for Generalized Orthogonal Matching Pursuit

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Abstract—Generalized orthogonal matching pursuit (gOMP), also called orthogonal multi-matching pursuit, is an extension of OMP in the sense that $N \geq 1$ indices are identified per iteration. In this paper, we show that if the restricted isometry constant (RIC) δ_{NK+1} of a sensing matrix \mathbf{A} satisfies $\delta_{NK+1} < 1/\sqrt{K/N+1}$, then under a condition on the signal-to-noise ratio, gOMP identifies at least one index in the support of any K -sparse signal \mathbf{x} from $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{v}$ at each iteration, where \mathbf{v} is a noise vector. Surprisingly, this condition does not require $N \leq K$ which is needed in Wang, *et al* 2012 and Liu, *et al* 2012. Thus, N can have more choices. When $N = 1$, it reduces to be a sufficient condition for OMP, which is less restrictive than that proposed in Wang 2015. Moreover, in the noise-free case, it is a sufficient condition for accurately recovering \mathbf{x} in K iterations which is less restrictive than the best known one. In particular, it reduces to the sharp condition proposed in Mo 2015 when $N = 1$.

Index Terms—Compressed sensing, restricted isometry constant, generalized orthogonal matching pursuit, support recovery.

I. INTRODUCTION

One of the central aims of compressed sensing is to recover a K -sparse unknown signal $\mathbf{x} \in \mathbb{R}^n$ (i.e., \mathbf{x} has at most K nonzero entries) from the following linear model [1] [2]

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{v}, \quad (1)$$

where $\mathbf{y} \in \mathbb{R}^m$ is an observation vector, $\mathbf{A} \in \mathbb{R}^{m \times n}$ (with $m \ll n$) is a given sensing matrix and $\mathbf{v} \in \mathbb{R}^m$ is a noise vector.

It has been shown that (see, e.g., [1]–[4]) stably recovering \mathbf{x} by some sparse recovery algorithms is possible under certain conditions on \mathbf{A} . One of the widely used frameworks for characterizing such conditions is the restricted isometry property (RIP) [1]. For a sensing matrix \mathbf{A} and for any integer K , the restricted isometry constant (RIC) δ_K of order K is defined as the smallest constant such that

$$(1 - \delta_K)\|\mathbf{x}\|_2^2 \leq \|\mathbf{A}\mathbf{x}\|_2^2 \leq (1 + \delta_K)\|\mathbf{x}\|_2^2 \quad (2)$$

for all K -sparse vectors \mathbf{x} .

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One of the most popular sparse recovery algorithms is orthogonal matching pursuit (OMP) [3]. Generalized orthogonal matching pursuit (gOMP) [5], also called orthogonal multi-matching pursuit [6], is an extension of OMP in the sense that $N(N \geq 1)$ indices are identified per iteration. Simulations in [5] and [6] indicate that, compared with OMP, gOMP has better sparse recovery performance. The gOMP algorithm is described in Algorithm 1, where \mathbf{A}_S denotes the submatrix of \mathbf{A} that contains only the columns indexed by set $S \subset \{1, 2, \dots, n\}$, \mathbf{x}_S denotes the subvector of \mathbf{x} that contains only the entries indexed by S . Note that when $N = 1$, gOMP reduces to OMP.

Algorithm 1 gOMP

Input: $\mathbf{y} \in \mathbb{R}^m$, $\mathbf{A} \in \mathbb{R}^{m \times n}$, K , $N \leq (m-1)/K$ and $\epsilon > 0$.
Initialize: $k = 0$, $\mathbf{r}^0 = \mathbf{y}$, $S_0 = \emptyset$.

1: **while** $k < K$ and $\|\mathbf{r}^k\|_2 > \epsilon$ **do**

2: $k = k + 1$

3: Choose indexes i_1, \dots, i_N corresponding to the N largest magnitude of $\mathbf{A}^T \mathbf{r}^{k-1}$,

4: $S_k = S_{k-1} \cup \{i_1, \dots, i_N\}$,

5: $\hat{\mathbf{x}}_{S_k} = \arg \min_{\mathbf{x} \in \mathbb{R}^{|S_k|}} \|\mathbf{y} - \mathbf{A}_{S_k} \mathbf{x}\|_2$,

6: $\mathbf{r}^k = \mathbf{y} - \mathbf{A}_{S_k} \hat{\mathbf{x}}_{S_k}$

7: **end while**

Output: $\hat{\mathbf{x}} = \arg \min_{\mathbf{x}: \Omega = S_k} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2$.

Many RIC-based conditions have been proposed to guarantee the accurately recovery of K -sparse signals with gOMP in the noise-free case (i.e., when $\mathbf{v} = 0$) for general N , such as $\delta_{NK} < 1/(\sqrt{K/N}+3)$ [5], $\delta_{NK} < 1/((2+\sqrt{2})\sqrt{K/N})$ [6], $\delta_{NK} < 1/(\sqrt{K/N}+2)$ and $\delta_{NK+1} < 1/(\sqrt{K/N}+1)$ [7]. Recently, it was further improved to $\delta_{NK} < 1/(\sqrt{K/N}+1.27)$ [8]. It is worthwhile pointing out that there are more sufficient conditions for OMP, see, e.g., [9]–[11].

Sufficient conditions of the exact support recovery of K -sparse signals with gOMP in the noisy case have also been widely studied (see e.g., [12] [13]). In particular, it was proved in [13] that under certain conditions on the minimum magnitude of the nonzero elements of \mathbf{x} , $\delta_{NK+1} < 1/(\sqrt{K/N}+1)$ is a sufficient condition under both ℓ_2 and ℓ_∞ bounded noises (i.e., $\|\mathbf{v}\|_2 \leq \epsilon$ and $\|\mathbf{A}^T \mathbf{v}\|_\infty \leq \epsilon$ for some constant ϵ , respectively).

In this paper, we aim to investigate RIP based sufficient conditions for the exact support recovery with gOMP in the noisy case. Instead of considering the ℓ_2 and ℓ_∞ bounded noises separately (see, e.g., [13]), we follow [14] and use the

signal-to-noise ratio (SNR) and the minimum-to-average ratio (MAR), which are respectively defined by

$$\text{SNR} = \begin{cases} \frac{\|\mathbf{A}\mathbf{x}\|_2^2}{\|\mathbf{v}\|_2^2} & \mathbf{v} \neq \mathbf{0} \\ +\infty & \mathbf{v} = \mathbf{0} \end{cases} \text{ and } \text{MAR} = \frac{\min_{i \in \Omega} |x_i|^2}{\|\mathbf{x}\|_2^2 / K}, \quad (3)$$

to measure \mathbf{v} and \mathbf{x} . The main reason that we use SNR is because it is a commonly used measure that compares the level of a desired signal to the level of background noise in science and engineering. We show that under a condition on SNR and MAR, gOMP is ensured to recover at least one index in the support of \mathbf{x} at each iteration if $\delta_{NK+1} < 1/\sqrt{K/N+1}$. As consequences, we have:

- Unlike [5] and [6], which require $N \leq \min(K, m/K)$, our condition on N is only $N \leq (m-1)/K$ which ensures that the assumption $\delta_{NK+1} < 1/\sqrt{K/N+1}$ makes sense. This allows more choices of N for gOMP.
- The exact support recovery condition for gOMP reduces to that for OMP when $N = 1$, and it is weaker than that proposed in [14] in terms of both SNR and RIP.
- In the noise-free case, we obtain that $\delta_{NK+1} < 1/\sqrt{K/N+1}$ is a sufficient condition for accurately recovering K -sparse signals with gOMP in K iterations. This improves the best known condition $\delta_{NK+1} < 1/(\sqrt{K/N}+1)$ [7]. Moreover, when $N = 1$, it is a sharp condition according to [10] [11].

The rest of the paper is organized as follows. We give some useful notation and lemmas in section II. We present our main results in Section III, and do numerical tests to illustrate them in Section IV. Finally, this paper is summarized in Section V.

II. NOTATION AND USEFUL LEMMAS

We introduce some notations and useful lemmas in this section.

A. Notation

Throughout this paper, we adopt the following notation unless otherwise stated. Let \mathbb{R} be the real field. Boldface lowercase letters denote column vectors, and boldface uppercase letters denote matrices. e.g., $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{m \times n}$. Let $\mathbf{0}$ denote a zero vector. Let Ω be the support of \mathbf{x} and $|\Omega|$ be the cardinality of Ω . Let set $S \subset \{1, 2, \dots, n\}$, and $\Omega \setminus S = \{i | i \in \Omega, i \notin S\}$. Let Ω^c and S^c be the complement of Ω and S , i.e., $\Omega^c = \{1, 2, \dots, n\} \setminus \Omega$, and $S^c = \{1, 2, \dots, n\} \setminus S$. Let \mathbf{A}_S be the submatrix of \mathbf{A} that only contains the columns indexed by S , and \mathbf{x}_S be the subvector of \mathbf{x} that only contains the entries indexed by S , and \mathbf{A}_S^T be the transpose of \mathbf{A}_S . For any full column rank matrix \mathbf{A}_S , let $\mathbf{P}_S = \mathbf{A}_S(\mathbf{A}_S^T \mathbf{A}_S)^{-1} \mathbf{A}_S^T$ and $\mathbf{P}_S^\perp = \mathbf{I} - \mathbf{P}_S$ denote the projector and the orthogonal complement projector on the column space of \mathbf{A}_S , respectively.

B. Useful lemmas

We now introduce some lemmas that will be used in the sequel.

Lemma 1 ([1]): If a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ satisfies the RIP of orders K_1 and K_2 with $K_1 < K_2$, then $\delta_{K_1} \leq \delta_{K_2}$.

Lemma 2 ([8]): Let S_1, S_2 be two subsets of $\{1, 2, \dots, n\}$ with $|S_2 \setminus S_1| \geq 1$. If a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ satisfies the RIP of order $|S_1 \cup S_2|$, then for any vector $\mathbf{x} \in \mathbb{R}^{|S_2 \setminus S_1|}$,

$$(1 - \delta_{|S_1 \cup S_2|}) \|\mathbf{x}\|_2^2 \leq \|\mathbf{P}_{S_1}^\perp \mathbf{A}_{S_2 \setminus S_1} \mathbf{x}\|_2^2 \leq (1 + \delta_{|S_1 \cup S_2|}) \|\mathbf{x}\|_2^2.$$

Lemma 3 ([15]): Let \mathbf{A} satisfy the RIP of order K and S be a subset of $\{1, 2, \dots, n\}$ with $|S| \leq K$, then for any $\mathbf{x} \in \mathbb{R}^m$, $\|\mathbf{A}_S^T \mathbf{x}\|_2^2 \leq (1 + \delta_K) \|\mathbf{x}\|_2^2$.

III. MAIN RESULTS

We propose our main results in this section. We begin with the following technical lemma.

Lemma 4: Let set $S \subseteq \{1, 2, \dots, n\}$ satisfy $|S| = kN$ and $|\Omega \cap S| = \ell$ for some integers N, k and ℓ with $0 \leq k \leq \ell \leq |\Omega| - 1$ and $N(k+1) + |\Omega| - k \leq m$. Let $W \subseteq \Omega^c$ satisfy $|W| = N$ and $W \cap S = \emptyset$. If \mathbf{A} in (1) satisfies the RIP of order $N(k+1) + |\Omega| - \ell$, then

$$\begin{aligned} & \max_{i \in \Omega \setminus S} |\mathbf{A}_i^T \mathbf{P}_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}| - \frac{1}{N} \sum_{j \in W} |\mathbf{A}_j^T \mathbf{P}_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}| \\ & \geq \frac{(1 - \sqrt{(|\Omega| - \ell)/N + 1} \delta_{N(k+1) + |\Omega| - \ell}) \|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{|\Omega| - \ell}}. \end{aligned} \quad (4)$$

Note that Lemma 4 extends [16, Lemma 1] for $N = 1$ to general N , and will play a key role in proving Theorem 1 below. Although it is motivated by [16, Lemma 1] and [11, Lemma II.2], it is stronger than [16, Lemma 1] and [11, Lemma II.2] since it holds for general N and for the noisy case (which contains the noise-free case as a special case). In contrast, [16, Lemma 1] is useful only when $N = 1$, and [11, Lemma II.2] is applicable only when $N = 1$ and $\mathbf{v} = \mathbf{0}$. In addition, regarding the proof itself, there are two key distinctions between Lemma 4 and [16, Lemma 1]. Due to the limitation of space, the proof of Lemma 4, the connections and differences between it and that of [16, Lemma 1] are detailed in the supplementary file.

Remark 1: The condition $N(k+1) + |\Omega| - k \leq m$ in Lemma 4 is to ensure the assumption that \mathbf{A} satisfies the RIP of order $N(k+1) + |\Omega| - \ell$ makes sense.

With Lemma 4, we can prove the following theorem.

Theorem 1: Let \mathbf{A} satisfy the RIP with

$$\delta_{N(k+1) + |\Omega| - k} < \frac{1}{\sqrt{|\Omega|/N + 1}} \quad (5)$$

for some integers k and N satisfying $0 \leq k \leq |\Omega| - 1$ and $N(k+1) + |\Omega| - k \leq m$. Then gOMP identifies at least one index in Ω in each of the first $k+1$ iterations until all the indexes in Ω are selected or gOMP terminates provided that

$$\sqrt{\text{SNR}} > \frac{\sqrt{2K}(1 + \delta_{N(k+1) + |\Omega| - k})}{(1 - \sqrt{|\Omega|/N + 1} \delta_{N(k+1) + |\Omega| - k}) \sqrt{\text{MAR}}}. \quad (6)$$

Proof: See Appendix B. ■

By Theorem 1 with $k = |\Omega| - 1$ and Lemma 1, we can obtain Theorem 2 below.

Theorem 2: Let \mathbf{A} satisfy the RIP with

$$\delta_{NK+1} < \frac{1}{\sqrt{K/N + 1}}, \quad (7)$$

for an integer N with $1 \leq N \leq (m-1)/K$. Then gOMP either identifies at least k_0 indexes in Ω if gOMP terminates after performing k_0 iterations with $1 \leq k_0 < K$ or recovers Ω in K iterations provided that

$$\sqrt{\text{SNR}} > \frac{\sqrt{2K}(1 + \delta_{NK+1})}{(1 - \sqrt{|\Omega|/N + 1}\delta_{NK+1})\sqrt{\text{MAR}}}. \quad (8)$$

When $N = 1$, gOMP reduces to OMP, and the following result can be directly obtained from Theorem 2.

Corollary 1: Let \mathbf{A} satisfy the RIP with $\delta_{K+1} < 1/\sqrt{K+1}$. Then OMP either identifies at least k_0 indexes in Ω if it terminates after performing k_0 iterations with $1 \leq k_0 < K$ or it recovers Ω in K iterations provided that

$$\sqrt{\text{SNR}} > \frac{\sqrt{2K}(1 + \delta_{K+1})}{(1 - \sqrt{|\Omega| + 1}\delta_{K+1})\sqrt{\text{MAR}}}. \quad (9)$$

Remark 2: The recovery condition for OMP in [14, Theorem 3.1] is

$$\delta_{K+1} < \frac{1}{\sqrt{K+1}}, \quad \sqrt{\text{SNR}} > \frac{2\sqrt{K}(1 + \delta_{K+1})}{(1 - (\sqrt{K} + 1)\delta_{K+1})\sqrt{\text{MAR}}}.$$

Clearly, our sufficient condition given by Corollary 1 is less restrictive than that given by [14, Theorem 3.1] in terms of both RIC and SNR.

Notice that gOMP may terminate after performing k_0 with $0 < k_0 < K$ iterations, and in this case Ω is not guaranteed to be recovered by gOMP under (7) and (8). However, we have:

Theorem 3: Suppose that $\mathbf{v} = \mathbf{0}$, and \mathbf{A} satisfies the RIP with (7) for an integer N with $1 \leq N \leq (m-1)/K$. Then gOMP recovers \mathbf{x} in K iterations.

Remark 3: In the noise-free case, the best known condition on δ_{NK+1} for accurately recovering \mathbf{x} with gOMP in K iterations is $\delta_{NK+1} < 1/(\sqrt{K/N} + 1)$ [7]. Obviously, our sufficient condition given by Theorem 3 is less restrictive.

Note that Theorem 3 can be directly obtained from Theorem 2 and Lemma 5 below.

Lemma 5: Suppose that $\mathbf{v} = \mathbf{0}$, and \mathbf{A} satisfies the RIP with (5) for some integers k and N with $1 \leq k \leq |\Omega| - 1$ and $1 \leq N \leq (m-1)/K$. If there exists an integer k_0 with $0 < k_0 \leq k$ and $|\Omega \cap S_{k_0}| \geq k_0$ such that $\|r^{k_0}\|_2 = 0$ (see Algorithm 1 for the definitions of S_{k_0} and r^{k_0}). Then $\Omega \subseteq S_{k_0}$.

Proof: We prove this lemma by contradiction. Suppose that $\Omega \not\subseteq S_{k_0}$ and let $\Gamma = \Omega \cup S_{k_0}$. Let $\bar{\mathbf{x}}, \tilde{\mathbf{x}} \in \mathbb{R}^{|\Gamma|}$ satisfy $\bar{x}_i = x_i$ for $i \in \Omega$ and $\bar{x}_i = 0$ for $i \notin \Omega$, and $\tilde{x}_i = (\hat{\mathbf{x}}_{S_{k_0}})_i$ for $i \in S_{k_0}$ and $\tilde{x}_i = 0$ for $i \notin S_{k_0}$, where $\hat{\mathbf{x}}_{S_{k_0}}$ is the vector generated by Algorithm 1. Since $\|r^{k_0}\|_2 = 0$, by line 6 of Algorithm 1, $\mathbf{A}_{S_{k_0}} \hat{\mathbf{x}}_{S_{k_0}} = \mathbf{y}$, we have

$$\mathbf{A}_\Gamma \bar{\mathbf{x}} = \mathbf{A}_\Omega \mathbf{x}_\Omega = \mathbf{A} \mathbf{x} = \mathbf{y} = \mathbf{A}_{S_{k_0}} \hat{\mathbf{x}}_{S_{k_0}} = \mathbf{A}_\Gamma \tilde{\mathbf{x}}. \quad (10)$$

Note that $|\Omega \cap S_{k_0}| \geq k_0$ and $\Gamma = \Omega \cup S_{k_0}$. Thus

$$|\Gamma| = |\Omega| + |S_{k_0}| - |\Omega \cap S_{k_0}| \leq |\Omega| + Nk - k \leq N(k+1) + |\Omega| - k.$$

By (5), \mathbf{A}_Γ is full column rank. Thus, applying (10) yields $\bar{\mathbf{x}} = \tilde{\mathbf{x}}$.

On the other hand, by the definitions of $\bar{\mathbf{x}}$ and $\tilde{\mathbf{x}}$, and the assumption that $\Omega \not\subseteq S_{k_0}$, there exists $j \in (\Omega \setminus S_{k_0})$ such that

$\bar{x}_j \neq 0$ but $\tilde{x}_j = 0$. This implies that $\bar{\mathbf{x}} \neq \tilde{\mathbf{x}}$ which contradicts with $\bar{\mathbf{x}} = \tilde{\mathbf{x}}$. Completing the proof. ■

Remark 4: When $N = 1$, Theorem 3 reduces to [11, Theorem III.1].

IV. NUMERICAL TESTS

In this section, we do numerical tests to illustrate Theorems 2 and 3. Since constructing general non-square deterministic matrices satisfying RIP with a given RIC is still an open problem, we use square sensing matrices to do tests. Specifically, for each given K and N , we assume $n = NK + 1$ and let $\mathbf{A} = \mathbf{D}\mathbf{U}$, where $\mathbf{D} \in \mathbb{R}^{n \times n}$ is a diagonal matrix with d_{ii} being uniformly distributed over $\left[\sqrt{1 - \frac{0.99}{\sqrt{K/N+1}}}, \sqrt{1 + \frac{0.99}{\sqrt{K/N+1}}}\right]$ for $1 \leq i \leq n$, and $\mathbf{U} \in \mathbb{R}^{n \times n}$ is an orthogonal matrix obtained by the QR factorization of a random matrix whose entries independent and identically follow the standard normal distribution. Then, by the definition of RIP, one can easily verify that \mathbf{A} satisfies the RIP with (7). For a given K , we generate a K -sparse vector $\mathbf{x} \in \mathbb{R}^n$. To illustrate Theorems 2 and 3, we respectively assume $\mathbf{v} = \frac{\|\mathbf{A}\mathbf{x}\|_2}{\sqrt{\text{SNR}}} \frac{\bar{\mathbf{v}}}{\|\bar{\mathbf{v}}\|_2}$ and $\mathbf{v} = \mathbf{0}$, where $\bar{\mathbf{v}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, and

$$\sqrt{\text{SNR}} = 0.01 + \frac{\sqrt{2K}(1 + \delta_{NK+1})}{(1 - \sqrt{|\Omega|/N + 1}\delta_{NK+1})\sqrt{\text{MAR}}}.$$

Note that MAR can be computed via (3) and $\delta_{NK+1} = \max\{1 - \min_{1 \leq i \leq n} d_{ii}, \max_{1 \leq i \leq n} d_{ii} - 1\}$. Clearly, (8) holds. After generating \mathbf{A}, \mathbf{x} and \mathbf{v} , \mathbf{y} can be computed via (1). Finally, we set $\epsilon = \|\mathbf{v}\|_2$ and use gOMP to recover \mathbf{x} . We did lots of tests by choosing different K and N and found that gOMP can always accurately recovering \mathbf{x} in the noise-free case and find its support in the noisy case.

V. CONCLUSION

In this paper, we have shown that under some conditions on SNR and MAR, $\delta_{NK+1} < 1/\sqrt{K/N+1}$ is a sufficient condition for the exact support recovery of K -sparse signals with gOMP. Surprisingly, unlike that in [5] and [6], this condition does not require $N \leq K$ which provides more choices for N . When $N = 1$, it is a sufficient condition for OMP and it is better than that proposed in [14]. In the noise-free case, it is a sufficient condition for accurately recovering K -sparse signals with gOMP in K iterations, which is better than the best known one in terms of δ_{NK+1} in [7]. Moreover, it reduces to the sharp condition in [11] when $N = 1$.

APPENDIX A PROOF OF LEMMA 4

In the following, we extend the proof of [16, Lemma 1] for $N = 1$ to general N to prove Lemma 4. Although our proof is highly relying on the techniques used in proving [16, Lemma 1] and [11, Lemma II.1], there are two main distinctions between these proofs. On the one hand, instead of defining a scalar t as in [16, Lemma 1] and [11, Lemma II.1], we need to define a vector $\bar{\mathbf{e}}_W \in \mathbb{R}^N$ (see (15)) to explore the fact that $|W| = N$. On the other hand, the choice of α (see

(13)) is also different. One can see from the following proof that both the well-defined $\bar{\mathbf{e}}_W$ and well-chosen α play a key role in proving Lemma 4.

Proof of Lemma 4. By [16, (21)], we have

$$\begin{aligned} & \|P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}\|_2^2 \\ & \leq \sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2 \max_{i \in \Omega \setminus S} |\mathbf{A}_i^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}|. \end{aligned} \quad (11)$$

In fact, since $|\Omega \cap S| = \ell \leq |\Omega| - 1$, $\|\mathbf{x}_{\Omega \setminus S}\|_1 \neq 0$. Thus, we obtain

$$\begin{aligned} & \max_{i \in \Omega \setminus S} |\mathbf{A}_i^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}| \\ & = \frac{1}{\|\mathbf{x}_{\Omega \setminus S}\|_1} \left(\sum_{j \in \Omega \setminus S} |x_j| \right) \max_{i \in \Omega \setminus S} |\mathbf{A}_i^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}| \\ & \stackrel{(a)}{\geq} \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} \left(\sum_{j \in \Omega \setminus S} |x_j| \right) \max_{i \in \Omega \setminus S} |\mathbf{A}_i^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}| \\ & \geq \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} \sum_{j \in \Omega \setminus S} (|x_j| \mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}) \\ & \geq \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} \sum_{j \in \Omega \setminus S} (x_j \mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}) \\ & = \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} \left(\sum_{j \in \Omega \setminus S} x_j \mathbf{A}_j \right)^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} \\ & = \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} (\mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S})^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} \\ & \stackrel{(b)}{=} \frac{1}{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2} \|P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}\|_2^2, \end{aligned}$$

where (a) follows from $|\text{supp}(\mathbf{x}_{\Omega \setminus S})| = |\Omega| - \ell$ and the Cauchy-Schwarz inequality, and (b) is from

$$(P_S^\perp)^T P_S^\perp = P_S^\perp P_S^\perp = P_S^\perp. \quad (12)$$

Thus, (11) holds.

Let

$$\alpha = -\frac{\sqrt{(|\Omega| - \ell)/N + 1} - 1}{\sqrt{(|\Omega| - \ell)/N}}, \quad (13)$$

then by one can easily verify that

$$\frac{2\alpha}{1 - \alpha^2} = -\sqrt{\frac{|\Omega| - \ell}{N}}, \quad \frac{1 + \alpha^2}{1 - \alpha^2} = \sqrt{\frac{|\Omega| - \ell}{N}} + 1. \quad (14)$$

To simplify notation, let $W = \{j_1, j_2, \dots, j_N\}$ and define $\bar{\mathbf{e}}_W \in \mathbb{R}^N$ with

$$(\bar{\mathbf{e}}_W)_i = \begin{cases} 1 & \text{if } \mathbf{A}_{j_i}^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} \geq 0 \\ -1 & \text{if } \mathbf{A}_{j_i}^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} < 0 \end{cases}, \quad 1 \leq i \leq N. \quad (15)$$

Then,

$$\bar{\mathbf{e}}_W^T \mathbf{A}_W^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} = \sum_{j \in W} |\mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}|. \quad (16)$$

Furthermore, define

$$\begin{aligned} \mathbf{B} &= P_S^\perp [\mathbf{A}_{\Omega \setminus S} \quad \mathbf{A}_W], \\ \mathbf{u} &= \begin{bmatrix} \mathbf{x}_{\Omega \setminus S} \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^{|\Omega \setminus S| + N}, \\ \mathbf{w} &= \begin{bmatrix} \mathbf{0} \\ \frac{\alpha \|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{N}} \bar{\mathbf{e}}_W \end{bmatrix} \in \mathbb{R}^{|\Omega \setminus S| + N}. \end{aligned} \quad (17)$$

Then,

$$\mathbf{B}\mathbf{u} = P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}, \quad (18)$$

and

$$\|\mathbf{u} + \mathbf{w}\|_2^2 = (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2, \quad (19)$$

$$\|\alpha^2 \mathbf{u} - \mathbf{w}\|_2^2 = \alpha^2 (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2. \quad (20)$$

Thus,

$$\begin{aligned} & \mathbf{w}^T \mathbf{B}^T \mathbf{B} \mathbf{u} \\ & \stackrel{(a)}{=} \frac{\alpha \|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{N}} \bar{\mathbf{e}}_W^T \mathbf{A}_W^T (P_S^\perp)^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} \\ & \stackrel{(b)}{=} \frac{\alpha \|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{N}} \bar{\mathbf{e}}_W^T \mathbf{A}_W^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S} \\ & \stackrel{(c)}{=} \frac{\alpha \|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{N}} \sum_{j \in W} |\mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}|, \end{aligned}$$

where (a) follows from (17)-(18); (b) follows from (12), and (c) is from (16). Therefore, we have

$$\begin{aligned} & \|\mathbf{B}(\mathbf{u} + \mathbf{w})\|_2^2 - \|\mathbf{B}(\alpha^2 \mathbf{u} - \mathbf{w})\|_2^2 \\ & = (1 - \alpha^4) \|\mathbf{B}\mathbf{u}\|_2^2 + 2(1 + \alpha^2) \mathbf{w}^T \mathbf{B}^T \mathbf{B} \mathbf{u} \\ & = (1 - \alpha^4) \left(\|\mathbf{B}\mathbf{u}\|_2^2 + \frac{2}{1 - \alpha^2} \mathbf{w}^T \mathbf{B}^T \mathbf{B} \mathbf{u} \right) \\ & = (1 - \alpha^4) (\|\mathbf{B}\mathbf{u}\|_2^2 \\ & \quad + \frac{2\alpha}{1 - \alpha^2} \frac{\|\mathbf{x}_{\Omega \setminus S}\|_2}{\sqrt{N}} \sum_{j \in W} |\mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}|) \\ & = (1 - \alpha^4) (\|\mathbf{B}\mathbf{u}\|_2^2 \\ & \quad - \frac{\sqrt{|\Omega| - \ell} \|\mathbf{x}_{\Omega \setminus S}\|_2}{N} \sum_{j \in W} |\mathbf{A}_j^T P_S^\perp \mathbf{A}_{\Omega \setminus S} \mathbf{x}_{\Omega \setminus S}|), \end{aligned} \quad (21)$$

where the last equality follows from the first equality in (14).

On the other hand, we have

$$\begin{aligned} & \|\mathbf{B}(\mathbf{u} + \mathbf{w})\|_2^2 - \|\mathbf{B}(\alpha^2 \mathbf{u} - \mathbf{w})\|_2^2 \\ & \stackrel{(a)}{\geq} (1 - \delta_{N(k+1)+|\Omega|-\ell}) \|\mathbf{u} + \mathbf{w}\|_2^2 \\ & \quad - (1 + \delta_{N(k+1)+|\Omega|-\ell}) \|(\alpha^2 \mathbf{u} - \mathbf{w})\|_2^2 \\ & \stackrel{(b)}{=} (1 - \delta_{N(k+1)+|\Omega|-\ell}) (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 \\ & \quad - (1 + \delta_{N(k+1)+|\Omega|-\ell}) \alpha^2 (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 \\ & = (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 [(1 - \delta_{N(k+1)+|\Omega|-\ell}) \\ & \quad - (1 + \delta_{N(k+1)+|\Omega|-\ell}) \alpha^2] \\ & = (1 + \alpha^2) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 [(1 - \alpha^2) - \delta_{N(k+1)+|\Omega|-\ell} (1 + \alpha^2)] \\ & = (1 - \alpha^4) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 (1 - \frac{1 + \alpha^2}{1 - \alpha^2} \delta_{N(k+1)+|\Omega|-\ell}) \\ & \stackrel{(c)}{=} (1 - \alpha^4) \|\mathbf{x}_{\Omega \setminus S}\|_2^2 (1 - \sqrt{(|\Omega| - \ell)/N + 1} \delta_{N(k+1)+|\Omega|-\ell}), \end{aligned} \quad (22)$$

where (a) follows from (17) and Lemma 2 (note that $|\Omega \cap S| = \ell$, $|W| = N$ and $|S| = kN$, leading to $|S \cup ((\Omega \setminus S) \cup W)| = N(k+1) + |\Omega| - \ell$), (b) follows from (19) and (20), and (c) follows from the second equality in (14).

By (18), (21), (22) and the fact that $1 - \alpha^4 > 0$, we have

$$\begin{aligned} & \|P_S^\perp A_{\Omega \setminus S} x_{\Omega \setminus S}\|_2^2 \\ & - \frac{\sqrt{|\Omega| - \ell} \|x_{\Omega \setminus S}\|_2}{N} \sum_{j \in W} |A_j^T P_S^\perp A_{\Omega \setminus S} x_{\Omega \setminus S}| \\ & \geq \|x_{\Omega \setminus S}\|_2^2 (1 - \sqrt{(|\Omega| - \ell)/N + 1} \delta_{N(k+1)+|\Omega|-\ell}). \end{aligned}$$

Thus, by (11), we obtain

$$\begin{aligned} & \max_{i \in \Omega \setminus S} |A_i^T P_S^\perp A_{\Omega \setminus S} x_{\Omega \setminus S}| - \frac{1}{N} \sum_{j \in W} |A_j^T P_S^\perp A_{\Omega \setminus S} x_{\Omega \setminus S}| \\ & \geq \frac{\|x_{\Omega \setminus S}\|_2 (1 - \sqrt{(|\Omega| - \ell)/N + 1} \delta_{N(k+1)+|\Omega|-\ell})}{\sqrt{|\Omega| - \ell}}. \end{aligned}$$

Therefore, Lemma 4 holds. \square

APPENDIX B PROOF OF THEOREM 1

We prove the result by induction. Suppose that gOMP selects at least one correct index in the first k iterations, then $\ell = |S_k \cap \Omega| \geq k$. We assume $\Omega \not\subseteq S_k$ (i.e., $\ell \leq |\Omega| - 1$) and Algorithm 1 performs at least $k+1$ iterations, otherwise, the result holds. Then, we need to show that $(S_{k+1} \setminus S_k) \cap \Omega \neq \emptyset$. Since $S_0 = \emptyset$, the induction assumption $|\Omega| > |S_k \cap \Omega| \geq k$ holds with $k = 0$. Thus, the proof for the first iteration is contained in the case that $k = 0$.

Let

$$W = \{j_1, j_2, \dots, j_N\} \subseteq \Omega^c \quad (23)$$

such that

$$|A_{j_1}^T r^k| \geq \dots \geq |A_{j_N}^T r^k| \geq |A_{j \in (\Omega^c \setminus W)}^T r^k|. \quad (24)$$

Then to show $(S_{k+1} \setminus S_k) \cap \Omega \neq \emptyset$, we only need to show

$$\max_{i \in \Omega} |A_i^T r^k| > |A_{j_N}^T r^k|.$$

By (24),

$$|A_{j_N}^T r^k| \leq \frac{1}{N} \sum_{j \in W} |A_j^T r^k|.$$

Thus, to show $(S_{k+1} \setminus S_k) \cap \Omega \neq \emptyset$, it suffices to show

$$\max_{i \in \Omega} |A_i^T r^k| > \frac{1}{N} \sum_{j \in W} |A_j^T r^k|. \quad (25)$$

By lines 4 and 5 of Algorithm 1, we have

$$\begin{aligned} r^k &= y - A_{S_k} \hat{x}_{S_k} = (I - A_{S_k} (A_{S_k}^T A_{S_k})^{-1} A_{S_k}^T) y \\ &\stackrel{(a)}{=} P_{S_k}^\perp (A x + v) \stackrel{(b)}{=} P_{S_k}^\perp (A_\Omega x_\Omega + v) \\ &= P_{S_k}^\perp (A_{\Omega \cap S_k} x_{\Omega \cap S_k} + A_{\Omega \setminus S_k} x_{\Omega \setminus S_k} + v) \\ &\stackrel{(c)}{=} P_{S_k}^\perp A_{\Omega \setminus S_k} x_{\Omega \setminus S_k} + P_{S_k}^\perp v, \end{aligned} \quad (26)$$

where (a), (b) and (c) follow from the definition of $P_{S_k}^\perp$, the fact that Ω is the support of x and $P_{S_k}^\perp A_{S_k} = 0$, respectively.

By lines 3 and 4 of Algorithm 1, for each $i \in S_k$, $|A_i^T r^k| = 0$. Thus, by (26) and the triangular inequality, we have

$$\begin{aligned} \max_{i \in \Omega} |A_i^T r^k| &\geq \max_{i \in \Omega \setminus S_k} (|A_i^T P_{S_k}^\perp A_{\Omega \setminus S_k} x_{\Omega \setminus S_k}| - |A_i^T P_{S_k}^\perp v|), \\ \frac{1}{N} \sum_{j \in W} |A_j^T r^k| &\leq \frac{1}{N} \sum_{j \in W} |A_j^T P_{S_k}^\perp A_{\Omega \setminus S_k} x_{\Omega \setminus S_k}| \\ &\quad + \max_{j \in W} |A_j^T P_{S_k}^\perp v|. \end{aligned}$$

(Note that instead of lower bounding $\max_{i \in \Omega} |A_i^T r^k|$ directly, it was first lower bounded by $\frac{\|A_{S_k}^T r^k\|}{\sqrt{N}}$, and then a lower bound on the latter quantity is given as a lower bound on $\max_{i \in \Omega} |A_i^T r^k|$ in [5, eq. (13)-(18)], this process requires $N \leq K$.) Thus, to show (25), it suffices to show

$$\beta_1 > \beta_2, \quad (27)$$

where

$$\begin{aligned} \beta_1 &= \max_{i \in \Omega \setminus S_k} |A_i^T P_{S_k}^\perp A_{\Omega \setminus S_k} x_{\Omega \setminus S_k}| \\ &\quad - \frac{1}{N} \sum_{j \in W} |A_j^T P_{S_k}^\perp A_{\Omega \setminus S_k} x_{\Omega \setminus S_k}|, \end{aligned} \quad (28)$$

$$\beta_2 = \max_{i \in \Omega \setminus S_k} |A_i^T P_{S_k}^\perp v| + \max_{j \in W} |A_j^T P_{S_k}^\perp v|. \quad (29)$$

In the following, we apply the technique used in the proof of [16, Theorem 1] to give an upper bound on β_2 . Clearly there exist $i_0 \in \Omega \setminus S_k$ and $j_0 \in W$ such that

$$\begin{aligned} \max_{i \in \Omega \setminus S_k} |A_i^T P_{S_k}^\perp v| &= |A_{i_0}^T P_{S_k}^\perp v|, \\ \max_{j \in W} |A_j^T P_{S_k}^\perp v| &= |A_{j_0}^T P_{S_k}^\perp v|. \end{aligned}$$

Therefore

$$\begin{aligned} \beta_2 &= \|A_{i_0 \cup j_0}^T P_{S_k}^\perp v\|_1 \stackrel{(a)}{\leq} \sqrt{2} \|A_{i_0 \cup j_0}^T P_{S_k}^\perp v\|_2 \\ &\stackrel{(b)}{\leq} \sqrt{2(1 + \delta_{N(k+1)+|\Omega|-k})} \|v\|_2, \end{aligned} \quad (30)$$

where (a) is because $A_{i_0 \cup j_0}^T P_{S_k}^\perp v$ is a 2×1 vector, (b) follows from Lemma 3, and

$$\|P_{S_k}^\perp v\|_2 \leq \|P_{S_k}^\perp\|_2 \|v\|_2 \leq \|v\|_2 \leq \epsilon.$$

In the following, we give a lower bound on β_1 . By line 3 of Algorithm 1, $|S_k| = kN$. By the induction assumption,

$$0 \leq k \leq |\Omega \cap S_k| = \ell \leq |\Omega| - 1. \quad (31)$$

By (23), $W \subset \Omega^c$ and $|W| = N$. Thus, by Lemmas 4 and 1, and (28), we obtain

$$\begin{aligned} \beta_1 &\geq \frac{(1 - \sqrt{(|\Omega| - \ell)/N + 1} \delta_{N(k+1)+|\Omega|-\ell}) \|x_{\Omega \setminus S_k}\|_2}{\sqrt{|\Omega| - \ell}} \\ &\geq \frac{(1 - \sqrt{|\Omega|/N + 1} \delta_{N(k+1)+|\Omega|-k}) \|x_{\Omega \setminus S_k}\|_2}{\sqrt{|\Omega| - \ell}}, \end{aligned} \quad (32)$$

where the second inequality follows from (31), the fact that $k \leq k$ and Lemma 1.

By [14, eq.(21)], we have

$$\|\mathbf{x}_{\Omega \setminus S_k}\|_2 \geq \sqrt{\frac{|\Omega| - \ell}{K(1 + \delta_{N(k+1)+|\Omega|-k})}} \sqrt{\text{MAR} \cdot \text{SNR}} \|\mathbf{v}\|_2. \quad (33)$$

In fact, by the fact that $\ell = |\Omega \cap S_k|$, we have

$$\begin{aligned} \|\mathbf{x}_{\Omega \setminus S_k}\|_2 &\geq \sqrt{|\Omega| - \ell} \min_{i \in \Omega} |x_i| \\ &\stackrel{(a)}{=} \sqrt{|\Omega| - \ell} \left(\sqrt{\text{MAR}} \|\mathbf{x}\|_2 / \sqrt{K} \right) \\ &\stackrel{(b)}{\geq} \sqrt{\frac{|\Omega| - \ell}{K(1 + \delta_{N(k+1)+|\Omega|-k})}} \sqrt{\text{MAR}} \|\mathbf{A}\mathbf{x}\|_2 \\ &\stackrel{(c)}{\geq} \sqrt{\frac{|\Omega| - \ell}{K(1 + \delta_{N(k+1)+|\Omega|-k})}} \sqrt{\text{MAR} \cdot \text{SNR}} \|\mathbf{v}\|_2, \end{aligned}$$

where (a) is from (3), (b) is from

$$\begin{aligned} \|\mathbf{A}\mathbf{x}\|_2 &= \|\mathbf{A}_\Omega \mathbf{x}_\Omega\|_2 \leq \sqrt{1 + \delta_{|\Omega|}} \|\mathbf{x}_\Omega\|_2 \\ &\leq \sqrt{1 + \delta_{N(k+1)+|\Omega|-k}} \|\mathbf{x}\|_2, \end{aligned}$$

and (c) follows from (3).

By (32) and (33), we have

$$\beta_1 \geq \frac{(1 - \sqrt{|\Omega|/N + 1} \delta_{N(k+1)+|\Omega|-k}) \sqrt{\text{MAR} \cdot \text{SNR}} \|\mathbf{v}\|_2}{\sqrt{K(1 + \delta_{N(k+1)+|\Omega|-k})}}.$$

Thus, by (30), (27) can be guaranteed by

$$\begin{aligned} &\frac{(1 - \sqrt{|\Omega|/N + 1} \delta_{N(k+1)+|\Omega|-k}) \sqrt{\text{MAR} \cdot \text{SNR}} \|\mathbf{v}\|_2}{\sqrt{K(1 + \delta_{N(k+1)+|\Omega|-k})}} \\ &> \sqrt{2(1 + \delta_{N(k+1)+|\Omega|-k})} \|\mathbf{v}\|_2, \end{aligned}$$

which is equivalent to (6). By induction, the theorem holds. \square

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